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PERFORMANCE OF DISTRIBUTED MULTI-ACCESS COMPUTER-COMMUNICATION SYSTEMS*

LEONARD KLEINROCK
University of California
Los Angeles, California

We consider an environment of geographically distributed message sources which communicate with each other over a "common" broadcast channel of fixed total capacity. This geographic dispersion along with the random generation times for messages creates a difficult channel resource-sharing problem. In this paper we discuss the performance degradation resulting from this problem, present known results for various multi-access schemes, define a measure of the "effective" number of users, demonstrate that "mixing" of access schemes cannot provide an improvement, introduce and analyze an effective dynamic reservation control access method, and finally present graphs showing the delay-throughput performance profiles in a fashion which displays the effect of the key system parameters.

1. INTRODUCTION

The technological advances which have provided remote access to computing facilities have also given rise to the technological problems of multi-access computer communications. Not only are we faced with analytic queueing problems arising from unpredictable message generation times and lengths, but we are also faced with the nasty issue of allocating a communications resource to a geographically distributed set of message sources. Were we not in this distributed environment, then queueing theory would provide us with the ultimate delay-throughput performance profiles; however, we have an additional loss due to the cost of organizing the separated sources into some kind of cooperating queue which permits intelligent access to the available channel bandwidth.

We consider an environment of M buffered message sources which are to share a communication channel of capacity C bits per second. Assume that the mth terminal (m=1,2,...,M) generates fixed length packets (of length b bits) according to a Poisson process at a rate λ_m messages per second. Thus the load placed upon the channel by this source is simply $\lambda_m b/C$ which we define to be S_m. The total normalized throughput may be expressed as $S=\Sigma S_m$. In queueing theory notation we have $S=\rho=\lambda b/C$, the utilization factor of the system, where $\lambda=\Sigma\lambda_m$.

We must now characterize the geographical distribution of these message sources. To simplify our task for purposes of analysis, we shall assume that we are in a packet switching broadcast radio communication environment where all terminals are separated by a propagation delay of d seconds which, when normalized, gives rise to the relative propagation delay $a=d/(b/C)$. We have now characterized the distributed computer communications problem in terms of the following parameters: M, $\{S_m\}$, S, and a. We are interested in evaluating the loss and performance capabilities of this system. Consider two important limiting cases. Specifically, if $a=0$, or if $M=1$, then we no longer have a distributed problem and the cost of creating a common queue disappears. Except for these two limiting cases (and the degenerate cases where all but one of the S_m's go to zero) we are faced with some loss in communications capacity which must be devoted to organizing the sources into a cooperative queueing structure. It is our goal to characterize the loss in the delay-throughput performance due to this distribution, and also to demonstrate an achievable performance profile in terms of a reduced set of system parameters.

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2. THE UNAVOIDABLE PRICE

As with most contention systems two factors contribute to a degradation in performance: first, there are the usual queueing effects due to the random nature of the message generation process; second, there is the cost due to the fact that our message sources are geographically distributed. If all the terminals were co-located (i.e., communications among them were free and instantaneous) then we could form a common queue of the generated message packets and achieve the optimum delay-throughput profile, namely, that of the M/D/1 queueing system [1] with a message input rate λ and a constant service time b/C seconds. Unfortunately we have M terminals which are distributed at a mutual normalized distance a and which independently generate messages. The total capacity we have available is C bits per second and we are faced with controlling access to this channel from these distributed message sources in which the control information must pass over the same channel which is being controlled (or over a control sub-channel which is derived from the data channel).

We have a spectrum of choices for introducing this control, ranging from no control at all to extremely tight static or dynamic control. For example, we could allow the terminals to access the channel using PURE (i.e., unslotted) ALOHA in which a terminal transmits a packet as soon as it is generated hoping that it will not collide with any other packet transmission; if there is a collision, then all packets involved in that collision are "destroyed" and must be retransmitted later at some randomly chosen time. This uncontrolled scheme is extremely simple, involves no control function or hardware, but extracts a price from the system in the form of wasted channel capacity due to collisions. At the other extreme, we could have a very tight fixed control as for example in FDMA or TDMA (see next section) where each terminal is assigned a sub-channel derived from the original channel. Such a fixed control scheme certainly avoids any collisions but is inefficient for two reasons: first, because terminals tend to be bursty sources and therefore much of their permanently assigned capacity will be wasted due to their high peak-to-average ratio; and second, the response time will be far worse in this channelized case due to the scaling effect which is especially apparent in FDMA (see [2]). A dynamic control scheme such as reservation-TDMA [3] (or Roberts' Reservation Scheme [4]) makes use of a reservation sub-channel through which terminals place requests for reserved space on the data channel; this system permits dynamic allocation of channel capacity according to a terminal's demand, but requires overhead in order to set up these reservations.

Thus we see that the issue of allocating capacity in a distributed environment is a serious one. In one form or another nature will extract her price! This price will appear in the form of collisions due to poor or no control, wasted capacity due to rigid fixed control, or overhead due to dynamic control. These comments are summarized in table 1 below.

Table 1: The price for distributed sources

	COLLISIONS	IDLE CAPACITY	OVERHEAD
No Control (e.g. ALOHA)	YES	NO	NO
Static Control (e.g. FDMA)	NO	YES	NO
Dynamic Control (e.g. Reservation Systems)	NO	NO	YES

We note that as M, the number of terminals, grows and as the geographical separation $\underline{a}$ grows, then also grows the price we pay for distribution. Given such a distributed terminal environment it would be interesting to determine exactly what is the minimum price in delay and throughput one must pay no matter what the access scheme; as yet no such result is known. Rather than such a lower bound, in this paper we provide an upper bound on the price one must pay.

3. A FAMILY OF MULTI-ACCESS METHODS

Multi-access methods have been evaluated in the past for distributed computer-communication systems. In this section we present the known results for a variety of these. We define T(S) to be the average time from when a packet is generated until it is successfully received. We are mainly concerned with the way in which the normalized* average response time T(S) varies with the overall system load S. It is perhaps best to think of all terminals transmitting to a central station which is the destination for these transmissions (this is not a necessary assumption since point-to-point communication also fits this model). S is the total system load which also represents the efficiency of channel use (as $S \to 1$, then useful data on the channel is being delivered at a rate of C bits per second; we neglect packet header overhead).

We now consider eight random multi-access schemes, and for each we give a reference and an extremely concise definition:

PURE (UNSLOTTED) ALOHA [5]: a newly generated packet will be transmitted by its terminal at the instant of its generation; collided packets destroy each other and must be retransmitted.

SLOTTED ALOHA [6,7]: the same as PURE ALOHA except that new packet transmissions must begin at the next slot point, where time is slotted into lengths equal to a packet transmission time.

CSMA (Carrier Sense Multiple Access) [6,8]: same as PURE ALOHA except that a terminal senses (listens to) the channel and can hear the carrier of any other terminal's transmission; if such a carrier is detected, then the terminal refrains from transmitting and follows one of many defined protocols.

POLLING [9]: a central controller sends a "polling message" to each terminal in turn; when a terminal is polled, it empties all of its data before indicating its empty buffer condition whereupon the next terminal is polled in sequence.

FDMA (Frequency Division Multiple Access) [9]: the bandwidth of the channel is divided into M equal sub-channels, each reserved for one of the M terminals.

TDMA (Time Division Multiple Access) [9]: time is slotted and a periodic sequence of the M integers is defined such that when a terminal's number is assigned to a slot, then that terminal (and only that terminal) may transmit in that slot; typically each terminal is given one out of every M slots.

MSAP (Mini-Slotted Alternating Priority [10]: a carrier-sense version of polling whereby a polling sequence is defined and when a terminal's buffer is empty, it simply refrains from transmitting; after $\underline{a}$ (normalized) time units, the next terminal in sequence senses the channel idle and proceeds with its transmission, etc. (This is also known as hub go-ahead polling.)

M/D/1 [1]: the classic first-come-first-serve single-server queueing system with Poisson arrivals and constant service time equal to a packet transmission time.

Of these eight previously studied schemes, the first three have rather complicated analytic expressions representing the delay-throughput performance. An approximation to their behavior will be given in section 4. For the others, we have the simple analytic expressions as follows (we assume $S_m = S/M$):

$$T_{POLL}(S) = \frac{2-S}{2(1-S)} + \frac{a}{2}\left(1 - \frac{S}{M}\right)\left[1 + \frac{M[2+(t_p/d)]}{1-S}\right] \quad (3.1)$$

$$T_{FDMA}(S) = M\left[\frac{2-S}{2(1-S)}\right] \quad (3.2)$$

$$T_{TDMA}(S) = 1 + M\left[\frac{1}{2} + \frac{S}{2(1-S)}\right] \quad (3.3)$$

$$T_{MSAP}(S) = \frac{2-S}{2(1-S)} + \frac{a}{2}\left(1 - \frac{S}{M}\right)\left(1 + \frac{M}{1-S}\right) \quad (3.4)$$

$$T_{M/D/1}(S) = \frac{2-S}{2(1-S)} \quad (3.5)$$

where t_p is the time to transmit a polling message. We note immediately that $T_{MSAP}(S) \le T_{POLL}(S)$ and so we will no longer consider polling in this discussion.

4. A THREE-PARAMETER APPROXIMATION

From the previous section we see that FDMA, TDMA and M/D/1 may all be approximated by delay functions of the form

$$T(S) = A\,\frac{Z-S}{P-S} \quad (4.1)$$

Here Z is the zero of the function, P is the pole of the function and A is a scalar multiplier. We require the following conditions: $AZ/P > 0$; $Z > P$ or $Z < 0$; $0 \le P \le 1$. It also turns out that MSAP is very nicely approximated by this function as long as $S \ll M$. For these access schemes we have:

ACCESS METHOD	A	Z	P
FDMA	M/2	2	1
TDMA	1	(2+M)/2	1
M/D/1	1/2	2	1
MSAP	(a+1)/2	[2+a(M+1)]/(1+a)	1

For analytic comparisons we may also express the ALOHA and CSMA access schemes by the (ZAP) approximation given above in eq. 4.1. For these three contention schemes we find the parameters A, Z and P as follows: first, we fit the pole of the function, thereby determining P; then we fit the value of the function at S=0, and last, in an ad-hoc fashion we fit the value of the function at some intermediate value of S in the vicinity of S=P/2. This third calculation permits us to modify the shape of the function, giving a rather nice approximation to many of the access schemes of interest.

In figs. 4.1, 4.2 and 4.3 below, we show the results of applying eq. 4.1 to seven access schemes for the cases M=10, M=100 and M=1000, respectively.

5. THE EFFECTIVE NUMBER OF USERS

As was pointed out in the author's previous paper at this Congress [2], there is a significant scaling effect for the response time in queueing systems. It states that if we had M systems, each with an input rate S/M and each with capacity C/M (with packet lengths of b bits) then a single system handling the total input rate S and with the total capacity C (a-

*T(S) is expressed in packet transmission times for a data channel whose capacity is C. That is, T(S) is normalized with respect to b/C seconds. Further, since all access methods require d seconds for propagation, we omit this additional delay term from all of our expressions. Thus $T(S) = [T_u(S)-d]C/b$, where $T_u(S)$ is the unnormalized average response time.

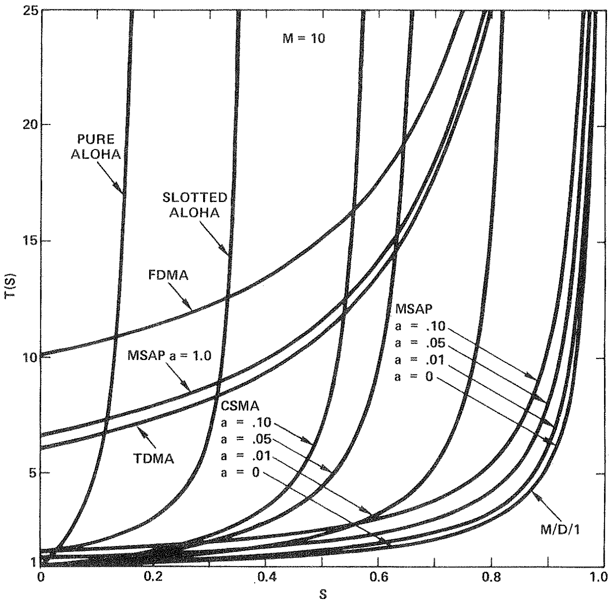

Fig. 4.1. Multi-access response times (M=10)

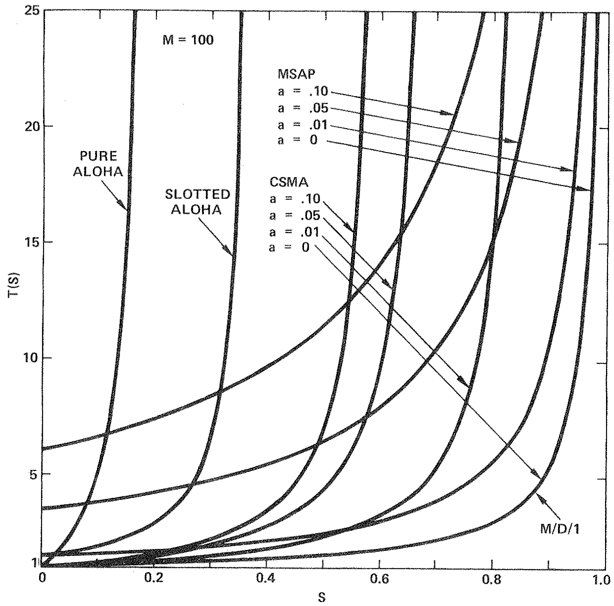

Fig. 4.2. Multi-access response times (M=100)

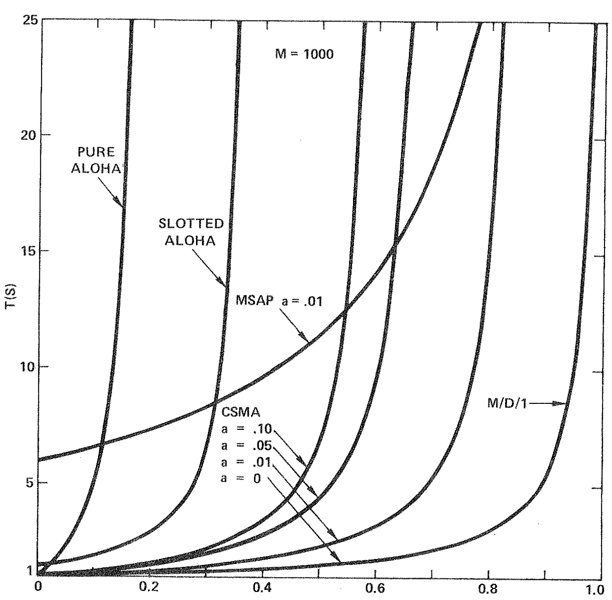

Fig. 4.3. Multi-access response times (M=1000)

gain with packets of length b bits) would have an average (unnormalized) response time T equal to 1/M that of each of the original systems. Another way to view this is to recognize that an FDMA access scheme serving M equal rate terminals will have an average response time which is M times as bad as a single M/D/1 system serving the entire population. Therefore it is natural for us to consider the ratio of the response time of a given access scheme and the response time which could be achieved in an M/D/1 system, to be a measure of the "effective number" of terminals in the system. Thus we define

$$M_{eff} \triangleq \frac{T_x(S)}{T_{M/D/1}(S)} \qquad (5.1)$$

where $T_x(S)$ is the mean response time for access method x. When x=FDMA and $S_m=S/M$, then $M_{eff}=M$. However, it should be clear that if the source rates were extremely unbalanced (say $S_1=S-(M-1)\varepsilon$, and $S_2=S_3=\ldots=S_M=\varepsilon$) then, as $\varepsilon\to 0$ we expect the system to behave as if there were only one terminal transmitting data. Our access scheme should take advantage of this fact. A natural way to allocate the total channel capacity C in this case of non-uniform source rates is to optimize each capacity so as to minimize the average delay; this exact problem was handled in [11] and the solution is simply the well-known "square root channel capacity assignment." Thus, with this optimum capacity assignment and for a given set M, $\{S_m\}$, S, a, we find that M^*_{eff} (the optimized M_{eff}) gives rise to a realistic performance measure and therefore an effective number of users given by

$$M^*_{eff} \triangleq \left(\sum_{m=1}^{M}\sqrt{\frac{S_m}{S}}\right)^2 \quad \text{(optimized FDMA)} \qquad (5.2)$$

Thus we are able to reduce the four-variable system description given in section 1 to a three-variable description M^*_{eff}, S, a, in which the first two variables of the original have merged into M^*_{eff}. Our problem is now reduced to evaluating the loss and performance of multi-access computer communications in the face of a number of users equal to M^*_{eff} which totally contribute a load S and which are geographically distributed at a "distance" a. Due to this simplification, we may, for the remainder of this paper, assume that the total load S is uniformly distributed among a number of users equal to $M^*_{eff}=M$; we will continue to study the ratio $T_x(S)/T_{M/D/1}(S)$ below.

6. MIXED ACCESS SCHEMES ARE NO GOOD

It is clear from figs. 4.1, 4.2 and 4.3 that for any given values of M, S, and a, there exists a best lower envelope along which the system could operate. We notice the sharp corners in this lower envelope which occur at the point where two competing access schemes intersect. One wonders if there is a method by which the two competing access schemes in the vicinity of this corner can share the load in some fashion so as to produce a performance which beats each one individually. Such is the subject of this section.

Let us consider any two access schemes which have delay-throughput functions that are nondecreasing functions of their arguments, that is

$$0 \le T_x(S_1) \le T_x(S_2) \qquad \text{for } S_1 \le S_2 \qquad (6.1)$$

We will assume that $T_1(0)>T_2(0)$ and $P_1>P_2$. For example, T_1 might represent the TDMA curve, whereas T_2 might represent the SLOTTED ALOHA curve in fig. 4.1. Let us assume that we are operating at a load of value $S=\lambda b/C$ on the curve T_1. Let us now introduce the "$\alpha\beta$ mix." What this mix accomplishes is to take a fraction α of the input traffic and a fraction β of the system capacity away from the first system and load it onto the second system, hopefully giving an improved performance. This means that the first system will be operating at a load $S_1=[\lambda(1-\alpha)b]/[C(1-\beta)]$ and that the second system will be operating at a load $S_2=\lambda\alpha b/C\beta$. That is, $S_1=(1-\alpha)S/(1-\beta)$ and $S_2=\alpha S/\beta$. In effect, what we have created are two subchannels, each handling a portion of the traffic and being allocated a portion of the capacity. For such a system, it is natural to define the response time as the appropriately weighted average of these two systems, where the weighting factor is proportional to the fraction of traffic which is handled by each. Moreover, we must be sure to renormalize the slot size to account for the change in channel capacity. That is, we define

$$T_{\alpha\beta}(S) = \frac{1-\alpha}{1-\beta}T_1(S_1) + \frac{\alpha}{\beta}T_2(S_2) \quad (6.2)$$

The question immediately arises as to what are the proper choices for α and β in the range $0\le\alpha\le1$, $0\le\beta\le1$. For example, it is clear that at S=0, we must have the values $\alpha=\beta=1$ yielding the minimum value $T_{1,1}(0)=T_2(0)$. Similarly, one should not be surprised if the optimum solution for α and β forces one to lie along a given access curve for a range of load values. What is not clear is whether there is a non-degenerate mix (i.e., $0<\alpha<1$, $0<\beta<1$) over some range of load values which gives performance superior to either of the two access methods alone. The answer is that such a range does not exist as we now prove.

THEOREM. The $\alpha\beta$ mix cannot improve the delay-throughput performance.

Proof: Let Q be the value of throughput at which the two access schemes have the same delay; that is

$$T_2(S) \le T_1(S) \qquad S\le Q \quad (6.3)$$

$$T_2(S) = T_1(S) \qquad S=Q \quad (6.4)$$

$$T_2(S) \ge T_1(S) \qquad S\ge Q \quad (6.5)$$

We are assuming that the two curves cross only once. If they cross more than once, the following argument may be repeated in each region and so our assumption costs us no loss in generality.

Our proof proceeds by assuming that mixing is good; we then show that this leads to a contradiction. We have two cases: $S\le Q$ and $S>Q$.

Case 1 $S\le Q$: If mixing is good, then there must be some values for α,β ($0<\alpha<1$, $0<\beta<1$) such that

$$T_{\alpha\beta}(S) = \frac{1-\alpha}{1-\beta}T_1\left(\frac{1-\alpha}{1-\beta}S\right) + \frac{\alpha}{\beta}T_2\left(\frac{\alpha}{\beta}S\right) < T_2(S) \quad (6.6)$$

Now, if $\alpha/\beta>1$, then $(\alpha/\beta)T_2(\alpha S/\beta)\ge T_2(S)$ by eq. 6.1 which contradicts eq. 6.6. Therefore $\alpha/\beta\le1$ which implies that $(1-\alpha)/(1-\beta)\ge1$. Therefore, by eqs. 6.1 and 6.3 we have

$$\frac{1-\alpha}{1-\beta}T_1\left(\frac{1-\alpha}{1-\beta}S\right) \ge T_1(S) \ge T_2(S)$$

which also contradicts eq. 6.6. Therefore mixing gives no improvement for $S\le Q$.

Case 2 $S>Q$: If mixing is good, then there must be some values for α,β ($0<\alpha<1$, $0<\beta<1$) such that

$$T_{\alpha\beta}(S) = \frac{1-\alpha}{1-\beta}T_1\left(\frac{1-\alpha}{1-\beta}S\right) + \frac{\alpha}{\beta}T_2\left(\frac{\alpha}{\beta}S\right) < T_1(S) \quad (6.7)$$

Now, if $(1-\alpha)/(1-\beta)>1$, then $\frac{1-\alpha}{1-\beta}T_1\left(\frac{1-\alpha}{1-\beta}S\right)\ge T_1(S)$ by eq. 6.1 which contradicts eq. 6.7. Therefore $(1-\alpha)/(1-\beta)\le1$ which implies that $\alpha/\beta\ge1$. Therefore by eqs. 6.1 and 6.5 we have

$$\frac{\alpha}{\beta}T_2\left(\frac{\alpha}{\beta}S\right) \ge T_2(S) \ge T_1(S)$$

which also contradicts eq. 6.7. Therefore mixing gives no improvement for $S>Q$.

Q.E.D.

7. A DYNAMIC RESERVATION SCHEME

We note from fig. 4.1 that the three contention schemes (ALOHA and CSMA) perform miserably as $S\to1$. The limiting behavior for M_{eff} as given in eq. 5.1 for the three schemes, FDMA, TDMA and MSAP is shown in the following table (recall that we now assume $S_m=S/M$):

	FDMA	TDMA	MSAP
$\lim_{S\to1} M_{eff}$	M	M	1+(M-1)a

Of course, the optimum value for M_{eff} is 1; FDMA and TDMA do poorly in this regard. MSAP also performs poorly in the case where Ma>>1; however, the more usual case is Ma<1 and so MSAP is not bad in the heavy traffic case. Nevertheless, it would be nice to design an access scheme which performed well in heavy traffic and which did not depend critically on the distribution parameter a. We define such an access method in this section.

Let us consider a dynamic control scheme based on reservations (not unlike Roberts' reservation scheme [4]). Assume that the total capacity C is divided into a request channel of capacity C_R and a data channel of capacity C_D such that $C_R+C_D=C$. We assume that before a terminal transmits any data, it must send a short control message over the channel for the purpose of placing a reservation on the data channel. The scheme we describe is not meant to be a practical scheme (although it could certainly be implemented), but rather to illustrate that dynamic control schemes do exist which have rather efficient behavior in the heavy traffic case. The request channel can operate under any access method; for purposes of this paper, we shall assume that the request capacity C_R is utilized among the M terminals in a simple TDMA fashion, thereby removing all contention effects. That is, each terminal is provided a private TDMA channel of capacity C_R/M. We further assume that both the request channel and the data channel are broadcast channels in the sense that all terminals can hear all transmissions When an idle terminal generates a packet, it first sends a request over the request channel. This request need be only one bit long, since activity on the channel itself identifies the terminal making the request; therefore, the average time required to transmit this one-bit request is simply $\bar{x}_R=(M+2)/2C_R$ seconds. Once the request is heard by all terminals, then it is known that this terminal has some data to send. Service on the data channel is given in a first-come-first-serve fashion, where the arrival time for a given terminal is defined to be the instant when its one-bit request is first heard on the request channel. So long as a terminal has not emptied its buffer, then its reservation status in the data channel queue is maintained in the sense that no new requests need be made until its buffer empties (as signalled perhaps by a bit in the last packet it transmits).

Let us calculate S_{Rm}, the load placed by the mth terminal on the request channel. Let $\bar{g}$ be the average time a terminal maintains its reserved status on the data channel. Using a simple approximation, it can be shown that $\bar{g}\ge g_L=\lambda(b/C_D)^2/[2(1-S_D)]$ where $S_D=SC/C_D$ is the load on the data channel. Once a terminal goes idle, the average time until it generates a new packet is M/λ seconds. Thus the average cycle time between requests on the request channel from a given terminal is $(M+2)/2C_R+\bar{g}+M/\lambda$. Finally, S_{Rm} is simply the request bit transmission time (M/C_R) divided by the average cycle time. The cycle time may be bounded from below by using $\bar{g}\ge g_L$, $(M+2)/2C_R+M/\lambda\ge0$ and $S_D\le1$ to give the upper bound $S_{Rm}\le2\lambda M(1-S_D)/C_RS_D$. We denote this upper bound by $\sigma\triangleq2\lambda M(1-S_D)/C_RS_D$. (We have "justified" the expression for σ by bounding S_{Rm}, but we could just as easily have pulled this definition "out of a hat.") Since $C=C_R+C_D$, $S_D=SC/C_D$ and $S=\lambda b/C$, we have

$$C_D = C\,\frac{\sigma b+2SM}{\sigma b+2M} \quad (7.1)$$

and therefore

$$C_R = C\,\frac{2M(1-S)}{\sigma b+2M} \quad (7.2)$$

From this last equation, we see the effect we were hoping for, namely, that the request channel capacity drops to zero as $S\to1$, thereby permitting the full capacity C to be used for the data channel in this heavy traffic case.

The average delay experienced by a packet can be shown to be the sum of the "set-up time" (i.e., the time to make the reservation over the request channel including an extra propagation delay of a normalized seconds) plus a regular queueing delay in an M/D/1 queueing system [10]. This second term is independent of the order of service due to the conservation law for queueing systems [6]. Thus our normalized response time is $T_{DYN}(S)=(M+2)C/2bC_R+(2-S_D)C/[2C_D(1-S_D)]+a$. If we now use eqs. 7.1 and 7.2, and the expression $S_D=SC/C_D$, we obtain

$$T_{DYN}(S) = \frac{1}{2(1-S)}\left(\frac{\sigma b+2M}{\sigma b}\right)\left[\sigma\frac{(M+2)}{2M} + \frac{(2-S)\sigma b+2MS}{\sigma b+2MS}\right] + a \qquad (7.3)$$

Let us now optimize the value of σ in the heavy traffic case. Factoring out the denominator term $2(1-S)$ and evaluating the remainder as $S\to1$, we have $\lim_{S\to1}2(1-S)T_{DYN}(S)=1+2M/\sigma b+\sigma(M+2)/2M+(M+2)/b$. If we differentiate this last expression, we find that the optimum value of σ which minimizes $T_{DYN}(S)$ at heavy traffic is $\sigma_{OPT}=2M/\sqrt{b(M+2)}$. This value may now be substituted back into eq. 7.3 to yield the performance of our dynamic scheme for all S as given by

$$T_{DYN}(S) = \frac{1}{2(1-S)}\left(1+\sqrt{\frac{M+2}{b}}\right)\left(1+\sqrt{\frac{M+2}{b}}\right) + \frac{1-S}{1+S\sqrt{\frac{M+2}{b}}} + a \qquad (7.4)$$

Of particular importance to us is the heavy traffic limit of the delay expression given earlier with the optimized σ; this yields $\lim_{S\to1}2(1-S)T_{DYN}(S)=[1+\sqrt{(M+2)/b}]^2$. We can now extend our earlier table to include the limiting form for M_{eff} for this dynamic scheme; we see that its behavior does not depend upon a at all and behaves very well as the message length b increases. In particular, this dynamic scheme will have a limiting value which is superior to MSAP so long as $a>a_0\triangleq2\sqrt{(M+2)/b}(M-1)+[(M+2)/(b(M-1))]$. For example, when M=10 and b=1000 we then require $a > .023$.

We shall use this scheme in the following section. Below in fig. 7.1, we graph the function $M_{eff}=T_{DYN}(S)/T_{M/D/1}(S)=[2(1-S)/(2-S)]T_{DYN}(S)$ for (M=10, b=1000), (M=100, b=1000) and (M=1000, b=100). It is obvious that unless M grows large with respect to b (i.e., M>>b), then the dynamic access method is quite excellent.

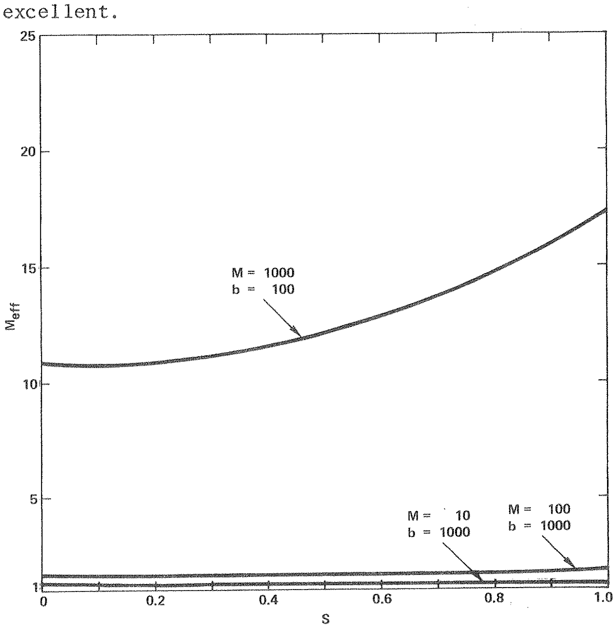

Fig. 7.1. Performance of the dynamic reservation scheme

8. ACHIEVABLE PERFORMANCE

Our purpose in this section is to display the many access schemes we have discussed (including the dynamic scheme of the previous section) by plotting $M_{eff}=T_x(S)/T_{M/D/1}(S)$ as a function of S. With this set of access schemes, and given the load parameter S, one can easily determine which among these many multi-access schemes is best; we simply get the lower envelope of the set of performance functions for a given value of a. This minimum envelope will always begin at the value $\min_x T_x(0)/T_{M/D/1}(0)$ where x varies over the set of access schemes. At the heavy traffic end, either MSAP or the dynamic scheme of the previous section will be optimal, and so we have

$$\min_x \frac{T_x(1)}{T_{M/D/1}(1)} = \begin{cases} 1 + a(M-1) & a<a_0 \\ \left(1+\sqrt{\frac{M+2}{b}}\right)^2 & a>a_0 \end{cases} \qquad (8.2)$$

where a_0 is the critical value of a separating the MSAP and the dynamic schemes, as given in the last section.

Our results produce the set of curves in figs. 8.1, 8.2 and 8.3 for (M=10, b=1000), (M=100, b=1000), and (M=1000, b=100), respectively. The minimum envelope represents performance which one can achieve in a distributed multi-access computer communication system for given values of M, S, and a. We note for M large that the ratio we are plotting departs from the optimum value of 1 in the middle and heavy load regions. At light loads we need give up no performance since we use a contention scheme whose price ordinarily is loss due to collisions; at light loads, however, we have

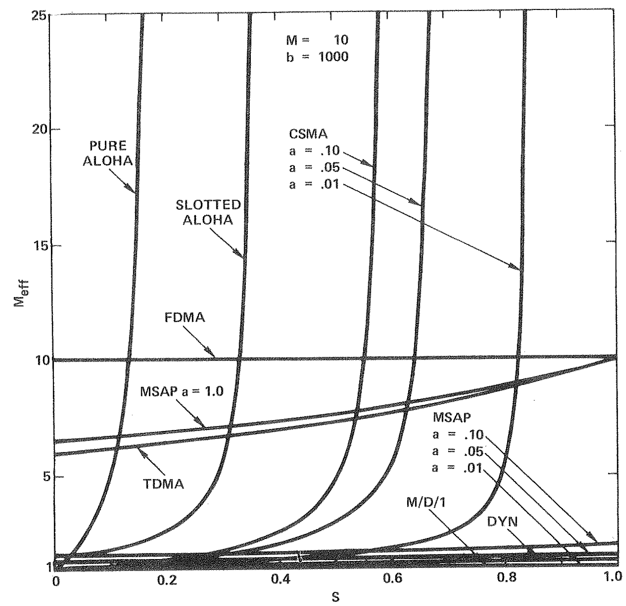

Fig. 8.1. Response time ratios (M=10, b=1000)

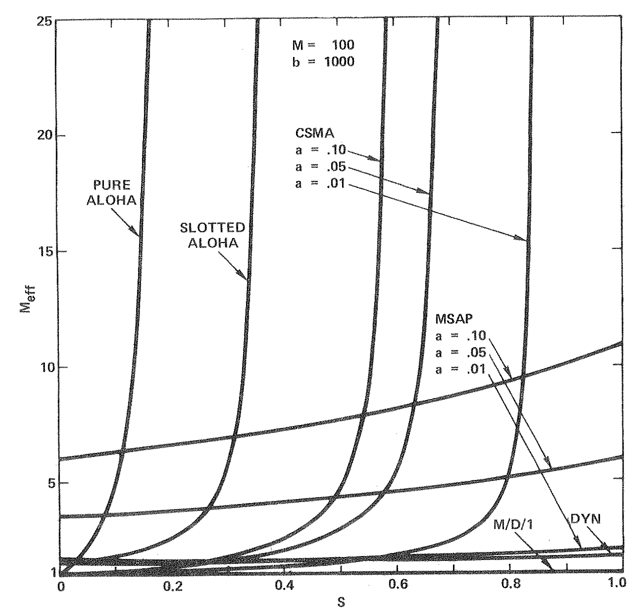

Fig. 8.2. Reponse time ratios (M=100, b=1000)

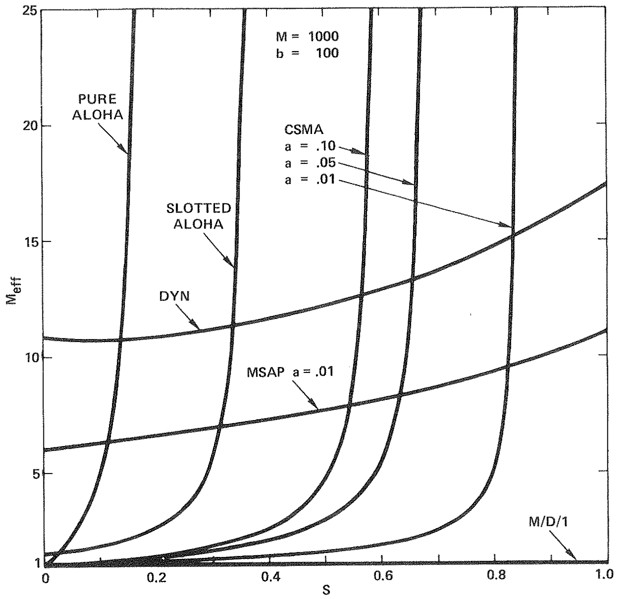

Fig. 8.3. Response time ratios (M=1000, b=100)

few collisions and therefore little loss. At heavy loads, we also find little loss compared to the optimum value of 1 so long as we use a controlled dynamic reservation scheme and M/b<<1; here too, the cost of control which ordinarily is overhead or empty slots, disappears due to the heavy load.

9. CONCLUSIONS

In this paper we have found it possible to characterize the behavior of distributed multi-access computer communication systems. We discussed the tradeoff between loss and performance, displayed some known multi-access schemes and gave reasonable approximations to their behavior, introduced M^*_{eff} which allowed us to reduce the system description to a three-parameter description, proved that an $\alpha\beta$ mix yields no improvement over the two access schemes being mixed, suggested a dynamic reservation scheme which performed very well at all loads and then finally in the last section were able to give a minimum envelope which selected the best known schemes available at various loads to produce an achievable performance. Thus we have effectively given an upper bound on delay (i.e., a lower bound on performance); this represents a loss in performance beyond which one need never go. It would be interesting to determine the upper bound on performance (i.e., the minimum loss which one must give up for any access scheme); such a bound is currently being worked on. One of the restrictive assumptions in this work was that all terminals were within "one hop" of each other, that is, all terminals could hear each others' transmissions perfectly. Of course, the more interesting case is that of a packet radio network in which some of the terminals are out of range of each other and must reach across the network by a series of repeaters. The characterization of loss and performance in such a network environment has yet to be carried out.

The cost of distributed sources must not only be paid in broadcast channels such as we have described here, but must also be paid in packet switching networks such as the ARPANET in which information regarding congestion and status is not available immediately, nor is it available for free. The adaptive routing algorithms which we often see in such networks represent the price we must pay both in overhead and delay and can be characterized in a fashion similar to that which was reported here.

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